Asymptotic Stability at Infinity for Differentiable Vector Fields of the Plane

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Abstract

Let $X : \mathbb{R}^2\backslash\overline{D}_\sigma \to \mathbb{R}^2$ be a differentiable (but not necessarily C^1) vector field, where $\sigma > 0$ and $\overline{D}_\sigma = \left\{z \in \mathbb{R}^2 : \|z\| \leq \sigma\right\}$. Denote by $\mathcal{R}(z)$ the real part of $z \in \mathbb{C}$. If for some $\epsilon > 0$ and for all $p \in \mathbb{R}^2\backslash\overline{D}_\sigma$, no eigenvalue of D_pX belongs to $(-\epsilon, 0] \cup \{z \in \mathbb{C} : \mathcal{R}(z) \geq 0\}$, then:
a) For all $p \in \mathbb{R}^2\backslash\overline{D}_\sigma$, there is a unique positive semi–trajectory of X starting at p;
b) It is associated to X, a well defined number $\mathcal{I}(X)$ of the extended real line $[-\infty, \infty)$ (called the index of X at infinity) such that for some constant vector $v \in \mathbb{R}^2$ the following is satisfied: if $\mathcal{I}(X)$ is less than zero (resp. greater or equal to zero), then the point at infinity ∞ of the Riemann sphere $\mathbb{R}^2 \cup \{\infty\}$ is a repellor (resp. an attractor) of the vector field $X + v$.

Key words: Planar vector fields; Asymptotic stability; Markus-Yamabe conjecture; Injectivity.

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